NOTATION

$X = x/L$ is the dimensionless coordinate; x is the coordinate, m; L is the thickness of the sample, m; $\theta = \lambda_0(t - t_0)/Lq_0$ is the dimensionless temperature; t is the sample temperature, K; t_0 is the temperature of the surround medium, K; λ_0 is the thermal conductivity of the standard, W/(m·K); q_0 is the amplitude of the heat flow oscillations, W/m^2; Fo is the Fourier number; Pe is the Peclet number; $\omega = \omega_0 L^2/a_0$ is the dimensionless angular frequency; ω_0 is the angular frequency of the heat flow oscillations, rad/sec; a_0 is the thermal diffusivity of the standard, m^2/sec; $\Lambda = \lambda/\lambda_0$ is the dimensionless thermal conductivity; λ is the thermal conductivity of the sample, W/(m·K); $A = a/a_0$ is the dimensionless thermal diffusivity; a is the thermal diffusivity, m^2/sec; $R = r_c\lambda_0/L$ is the dimensionless thermal resistance; r_c is the contact thermal resistance of the gap, m^2·K/W; $C = C_c h a_0/\lambda_0 L$ is the dimensionless heat capacity; C_c is the heat capacity of the contact gap, J/(m^3·K); and L^* is the characteristic dimension of the structure of the porous medium, m.

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PERMEABILITY AND PERCOLATIONAL PROPERTIES OF SEDIMENTARY ROCK

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UDC 532.546

Formulas for calculating the absolute and relative phase permeabilities of rock are obtained, taking account of the percolational properties of the pore space.

INTRODUCTION

The absolute and relative phase permeabilities are among the most important characteristics of sedimentary rock, and are determined primarily by the structure of the pore space.

Until recently, a simple model consisting of a bundle of parallel noninteracting capillaries was widely used to calculate the permeability. Considerable effort has been expended in trying to understand the factors responsible for the rock permeability and to eliminate the deficiencies of the simple capillary model. Percolation-theory concepts and methods, taking account of the coupling between different capillaries, have been of particular importance here [1-6].

In the usual formulation of the percolation-theory problem for the description of processes occurring in rock, the pore space is represented as large pores (points) and thin channels (bonds connecting the pores) [2]. The permeability in this lattice is determined basically by the thin channels; with decrease in their number, the interconnection of the pore space is disrupted and fluid filtration is impossible.

In recent years, the idea that there is no closed porosity in collector rock (at least, in terrestrial rock) [7]. Hence it follows that, with any porosity values, there is an interconnected pore space through which the flow of various fluids is possible [8]. Therefore, the filtrational properties must evidently be associated not with the geometric coupling of the pore space but with the coupling of the conducting pores.

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It was concluded in [10] that the conductivity of very disordered media with a broad distribution of local conductivities is determined by the elements with the appropriate conductivities which first create the flow, if the elements of the medium are connected in series in the order of increasing resistance. Then the permeability problem reduces to the percolational problem with a threshold permeability value k_c.

In [1], analogously to [11], an expression for the permeability was obtained

$$k \sim r_c^2 \varepsilon_i, \tag{1}$$

where ε_i is the volume fraction of pores with $r_i \geq r_c$.

In deriving Eq. (1), it is assumed that, after the appearance of an infinite cluster of filtrational pores, the addition of new groups of pores of radius $r_i < r_c$ will have little influence on the permeability. In reality, because of the continuity of the size distribution function of the pores, the addition of new pores considerably increases the permeability [6].

The concept of [10] was further developed in [12, 13]. All the elements of the medium with permeability $k \geq k_c$ are assigned the value k_c, and those with $k < k_c$ are assigned zero permeability. As a result, the following equation for k is obtained

$$k = ak_c(p(k_c) - p_c)^t, \tag{2}$$

where a is a constant; $p(k_c)$ is the probability that a given permeability will be greater than or equal to k_c; $t = 1.9$ for a three-dimensional lattice.

In [14], numerical modeling for a two-dimensional lattice confirmed the quantitative accuracy of Eq. (2) and the conclusions of [10] for various conductivity distribution laws.

In [15], a very successful recent work, the concepts of [10-12] were used to calculate the absolute permeability on the basis of the mercury depression curve. In an approach taking account of the most important conducting paths in calculating k, the following formula for the absolute permeability was derived in [15]

$$k = (1/89)\,(L_{\max}^h)^2\,(L_{\max}^h/L_c)\,mS\,(L_{\max}^h), \tag{3}$$

where L_c is the pore breakdown size, determined from the mercury depression curve; $L_{\max}{}^h$ is the optimal pore size for calculating the permeability, when the function f(L), which is the volume of depressed mercury multiplied by L^3, is a maximum.

The parameter in Eq. (3) which is the most difficult to determine is $L_{\max}{}^h$, the physical meaning of which is not obvious.

1. FORMULAS FOR THE ABSOLUTE AND RELATIVE PHASE PERMEABILITIES

The absolute and relative phase permeabilities of the rock are calculated using the capillary-pressure curves obtained by the semipermeable-membrane method or the mercury depression method, taking account of the percolational properties of the pore space. In contrast to [15], a different scheme is used to determine the characteristic permeability k_c.

The critical permeability k_c is defined by the expression

$$k_c = \frac{m}{8} \int_{r_0}^{r_c} r^2 \left(\frac{r}{r_c}\right) f(r)\,dr, \tag{4}$$

where r_0 is the pore radius below which the pores are nonfiltrational; f(r) is the density function of the pore size distribution; r_c corresponds to the condition of formation of an infinite pore cluster with $r > r_c$ in the semipermeable-membrane method or the mercury-depression method in a rock sample.

Rewriting Eq. (4) in terms of the capillary-pressure curve [4], this curve is integrated from the residual saturation $S_{nw}{}^0$ of the nonwetting phase to the maximum saturation S^*_{nw} obtained by the semipermeable-membrane method or the mercury-depression method. Then, taking $\cos\theta \sim 1$

$$k_c \sim \frac{m\sigma^2}{2}\,p_c \int_{S_{nw}^0}^{S_{nw}^*} \frac{dS_{nw}}{p_{ca}^3(S_{nw})}, \tag{5}$$

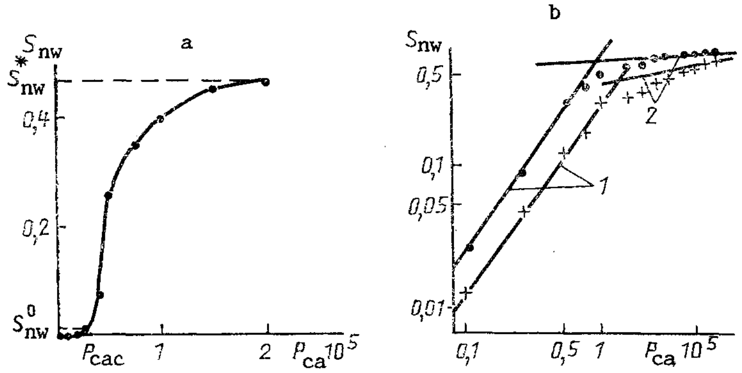

Fig. 1. Capillary curves. p_{ca}, Pa.

where $p_{ca}(S_{nw})$ is the capillary pressure.

To calculate the permeability, it remains to determine the probabilities p and p_c in Eq. (2). As is known, the problem of the displacement corresponding to the mercury depression in the sample and the displacement of water by air in the semipermeable-membrane method may be formulated in terms of percolation theory. In this case, the onset of filtration of the nonwetting phase is equivalent to the formation of an infinite cluster of filtrational pores of the sample [17-19]. The infinite cluster exists when $p_{ca} > p_{cac}$; the asymptotic behavior of the volume fraction of nonwetting phase S_{nw} close to the percolation threshold, i.e., at small $\Delta p_{ca} = p_{ca} - p_{cac} > 0$, is as follows

$$S_{nw} \sim \Delta p_{ca}^{\beta} \tag{6}$$

where β is the index for the probability that a filled pore belongs to the infinite cluster.

A characteristic capillary-pressure curve obtained by the semipermeable-membrane method is shown in Fig. 1a in the coordinates p_{ca} and S_{nw}. The actual data are denoted by points. The characteristic saturation values of the nonwetting phase $S_{nw}{}^0$ and S^*_{nw} on the $pca(S_{nw})$ curve are denoted by dashed lines. To find β, the quantity $S_{nw}{}^0$ which does not participate in the formation of an infinite pore cluster but fills the dead-end pores appearing at the end of the sample may be subtracted from S_{nw}. In addition to the foregoing, note that Eq. (2) is universal with respect to t, i.e., t does not depend on the properties of the medium at the micro level, but only on the large-scale properties of the percolational system, which is expressed in that t only depends on the dimensionality of the system.

This universality is disrupted when the permeability (or conductivity) at the micro level depends on the geometric properties, for example, the channel radius: $k \sim r^n$. It follows from the relation f(k)dk = h(r)dr that f(k) = h(r)dr/dk. For media with a density function of the probability distribution of the form $f(k) \sim k^{-\alpha}$ $(0 < \alpha < 1)$, the expression for the permeability is [20]

$$k \sim (p - p_c)^{t+\alpha/(1-\alpha)}. \tag{7}$$

Using Eqs. (5)-(7), the absolute permeability is found in the form

$$k \sim \frac{m\sigma^2}{2} p_c \int_{S^0_{ca}}^{S^*_{nw}} \frac{dS_{nw}}{p^3_{\text{к}}(S_{ca})} (S^*_{nw} - S^0_{nw})^{\bar{t}}, \tag{8}$$

where $\bar{t} = [t + \alpha/(1 - \alpha]/\beta$. For the case of capillary tubes, $k \sim r^3$, i.e., $\alpha = 2/3$.

The relative phase permeabilities for water $k_W(S)$ and petroleum $kp(S)$ in the case of drainage (displacement of wetting phase by nonwetting phase) may be calculated from the following formulas, under the assumption of jet flow of the phases [8, 17]

$$k_W(1 - S_{nw}) \sim \frac{m\sigma^2}{2} p_c \int_{S_{nw}}^{S^*_{nw}} \frac{dS_{nw}}{p^3_{ca}(S_{nw})} (S^*_{nw} - S_{nw})^{\bar{t}} / k, \tag{9}$$

$$k_P(1 - S_{nw}) \sim \frac{m\sigma^2}{2} p_c \int_{S^0_{nw}}^{S_{nw}} \frac{dS_{nw}}{p^3_{ca}(S_{nw})} (S_{nw} - S^0_{nw})^{\bar{t}} / k. \tag{10}$$

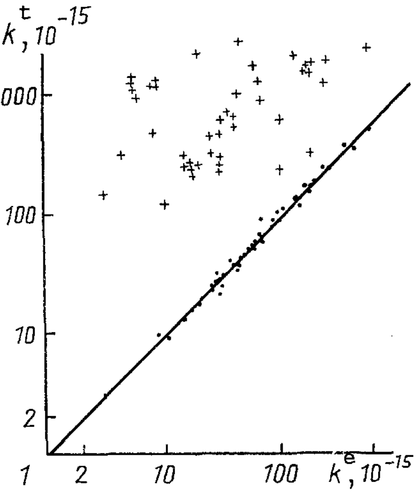

Fig. 2

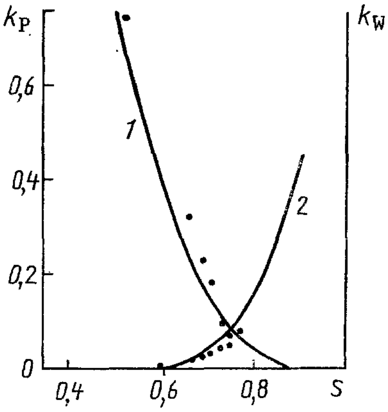

Fig. 3

Fig. 2. Comparison of theoretical k^t and experimental k^e permeability values. k^t, k^e, m^2.

Fig. 3. Curves of relative phase permeability with respect to nonwetting (1) and wetting (2) phases.

In contrast to the formulas of [21], which contain parameters that cannot be determined directly from experiment, Eqs. (8)-(10) permit the calculation of the absolute and relative phase permeabilities in the presence of capillary-pressure curves.

2. COMPARISON OF THEORETICAL AND EXPERIMENTAL PERMEABILITIES

The absolute and relative phase permeabilities are calculated for samples of sandstone and aleurite from the Komsomol, Tyansk, and Samotlorsk deposits. Capillary-pressure curves $S_{nw}(p_{ca})$ have been obtained by the semipermeable-membrane method at TsL Tyumen'geologiya and VNIGIK for samples from the first two deposits and by the mercury-depression method at the All-Union Scientific Research Institute of Natural Gas for the Samotlorsk samples.

Experimental data obtained by the semipermeable-membrane method for two rock samples with absolute permeability $20 \cdot 10^{-15}$ m^2 (crosses) and $96 \cdot 10^{-15}$ m^2 (points) are shown in Fig. 1b in the logarithmic coordinates $S_{nw}-p_{ca}$. With gradual increases in pressure, the saturation of the nonwetting phase increases to some characteristic value $S_{nw}{}^0$ (Fig. 1a), at which a coupled pore cluster appears. Until the front reaches the end of the sample, the dependence $S_{nw}(p_{ca})$ is described by curve 1; then the finite size of the sample begins to influence the displacement, and the experimental data are approximated by curve 2.

Similar behavior was observed in a numerical experiment on the diffusional front in the mutual diffusion of two types of particles in a square lattice, when the front reaches one boundary [22].

In Fig. 2, theoretical values of the absolute permeability from Eq. (8) (points) are compared with experimental data and the results of the simple capillary model (crosses). In the calculations, a surface tension of 70 mN/m is assumed [23]. Taking account of the accuracy in measuring the permeability and in determining p_c and $S_{nw}{}^0$, as well as the assumption $a \sim 1$, the agreement of Eq. (8) with experiment may be regarded as satisfactory. The values of k obtained by the capillary model, disregarding the percolational characteristics of the pore space, are considerably higher than the experimental data.

The curves for the relative phase permeabilities for drainage obtained on the basis of Eqs. (9) and (10) agree with the analogous curves for experimental data on steady filtration. The curves in Fig. 3 show the relative phase permeabilities obtained from Eqs. (9) and (10) for sandstone of the Tyansk deposit with $k = 19.1 \cdot 10^{-15}$ m^2, while the points correspond to experimental data on kp and k_W [24] obtained for a Samotlorsk sample with $k = 22.6 \cdot 10^{-15}$ m^2 by the steady-filtration method.

CONCLUSIONS

1. Simple formulas have been obtained for the absolute and relative phase permeabilities of rock samples using the capillary-pressure curves without fitting parameters.

2. It has been shown that calculation of the permeability from the simple capillary model, disregarding the percolational properties of the pore space, leads to incorrect results.

3. The exponent in the expression for the permeability is not universal. It depends on the structural features of the pore space and the conducting properties of its component parts.

NOTATION

k, k_w, k_p, k_c, absolute and relative phase permeabilities and characteristic permeability; r, pore radius; r_c, breakdown pore radius; $S(L)$, proportion of coupled pore space formed by pores of radius L or more; m, porosity; σ, surface tension; S, S_{nw}, saturation of pure space of wetting and nonwetting phases; t, conductivity index.

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